Math 72	9.2 – 1st	Evaluate exponentials Graph Exponentials
	9.2 2nd	Solve Exponential Equations.
Math 62	11.2 – 1st	
	11.2 - 2nd	

Objectives

1) Evaluate exponential functions
2) Graph exponential functions
3) Solve exponential equations by the "same base" method.

Math 70 12.2 Exponential Functions, Exponential Equations, and Word Problems

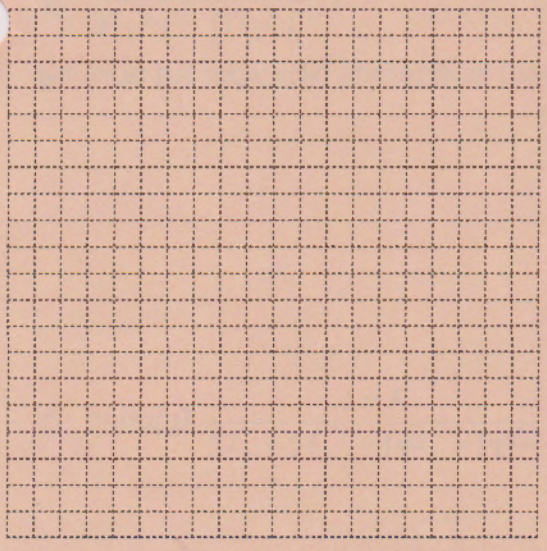
Objectives:

1. Graph exponential functions.
2. Solve exponential equations algebraically.
3. Solve word problems that result in exponential equations.

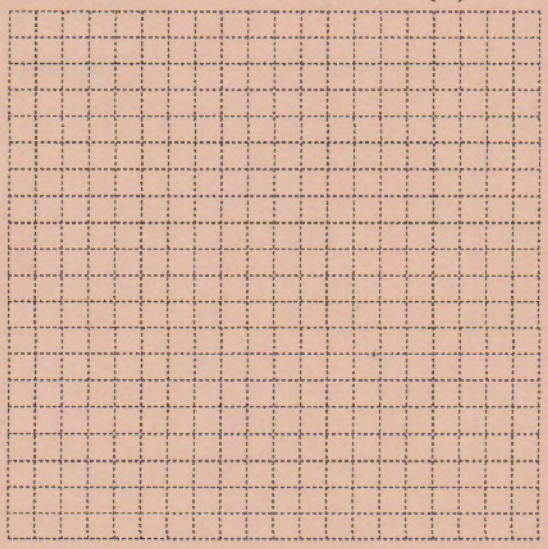
1) Evaluate $f(x) = 2^x$ to complete the following table:

x	$f(x) = 2^x$
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

2) Sketch the graph of $f(x) = 2^x$



3) Sketch the graph of $f(x) = \left(\frac{1}{3}\right)^x + 2$



When sketching the graphs of exponential functions:

1. Plot enough points to show exponential growth.
2. Plot the y-intercept
3. Plot enough points to show the horizontal asymptote.

Solve the equation.

4) $2^x = 16$

5) $3 \cdot 2^x = 48$

6) $2^{3x+1} = 16$

7) $2 \cdot 9^x = 54$

8) $4^{x+3} = 8^x$

9) $\frac{5}{4} = 5 \cdot 2^{3x}$

10) Use GC to estimate the solution to $5^x = 10$

11) Find the amount owed at the end of 5 years if $3000 is loaned at a rate of 10% compounded quarterly.

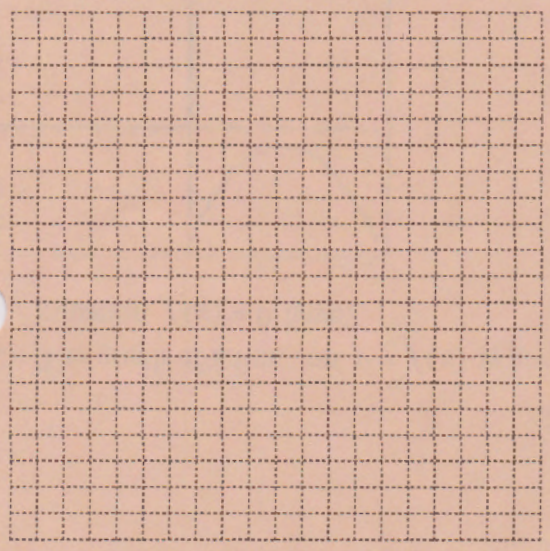
12) An unusually wet spring has caused the size of the Cape Cod mosquito population to increase by 8% each day. If an estimated 200,000 mosquitoes are on Cape Cod on May 12, find how many mosquitoes will inhabit the Cape on May 25. Use $y = 200{,}000(2.7)^{0.08t}$, where y is the number of mosquitos and t is the number of days since May 12. Round to the nearest thousand.

EXTRAS:

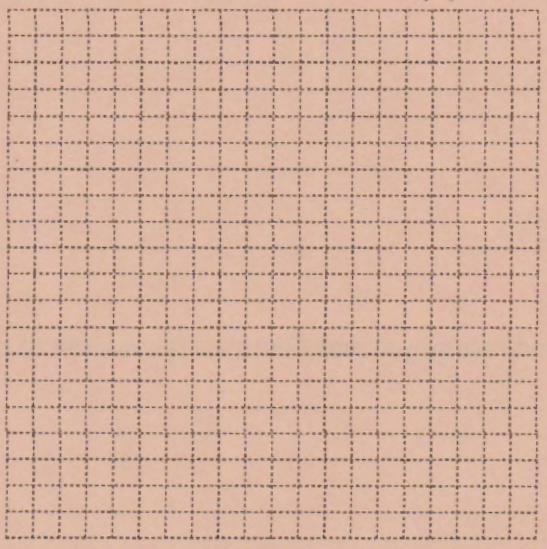
13) Evaluate $f(x) = 3^x$ to complete the following table:

x	$f(x) = 3^x$
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

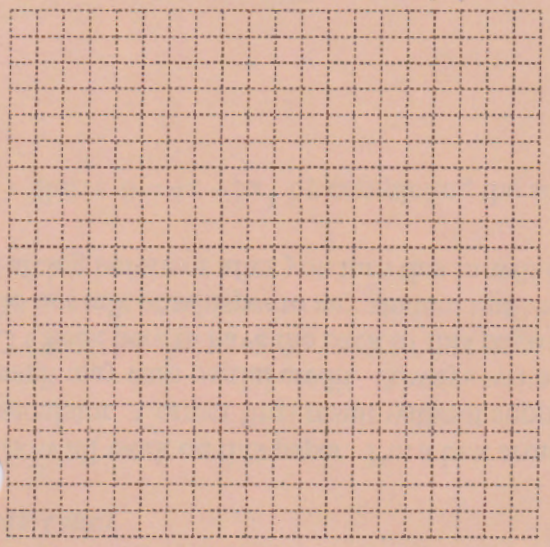
14) Sketch the graph of $f(x) = 3^x$



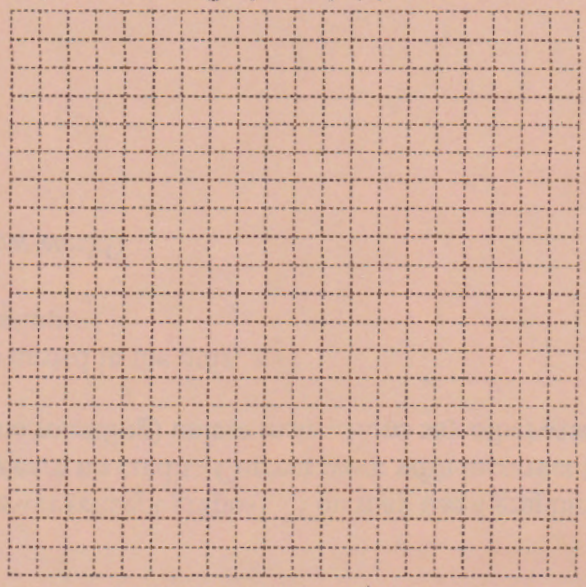
15) Sketch the graph of $f(x) = \left(\frac{1}{3}\right)^x$



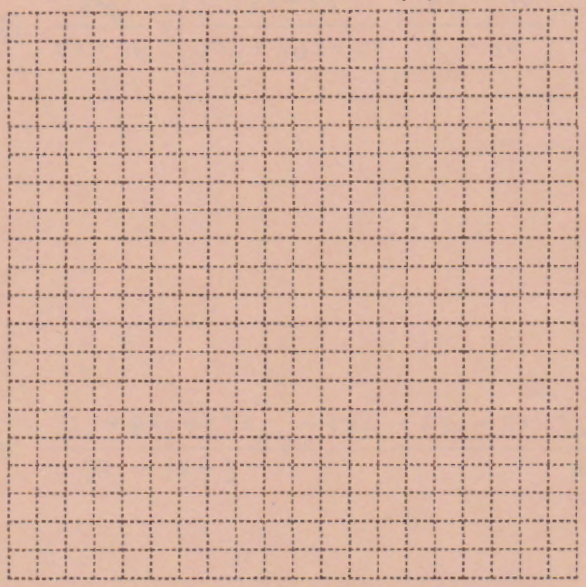
16) Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$



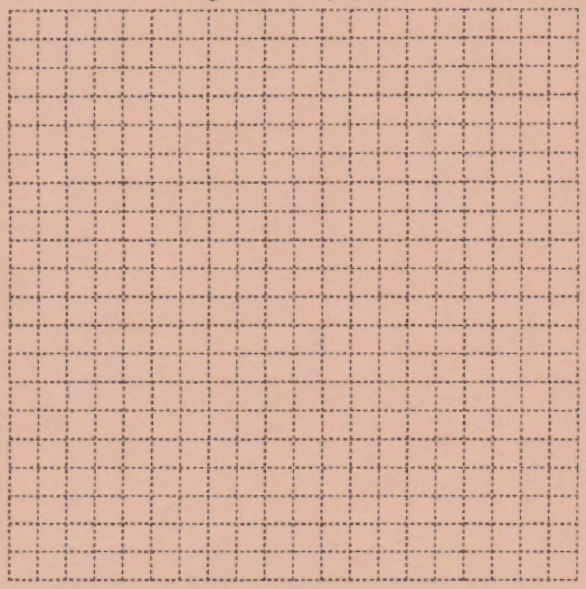
17) Sketch the graph of $f(x) = 3^{x+2}$



18) Sketch the graph of $f(x) = -\left(\frac{1}{5}\right)^x$



19) Sketch the graph of $f(x) = -2^x$



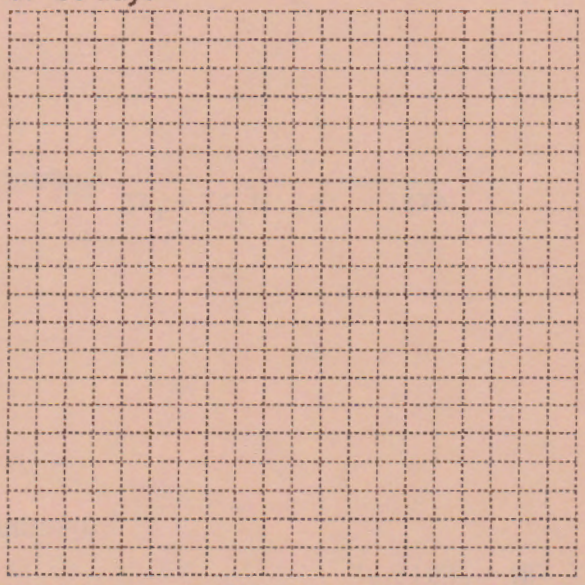
Solve the equation.

20) $25^x = \frac{1}{5}$

21) $4^{3x-7} = 32^{-2}$

22) As a result of the Chernobyl nuclear accident, radioactive debris was carried through the atmosphere, affecting cows and the milk they provide. The percent *y* of radioactive material in raw milk after *t* days is estimated by $y = 100(2.7)^{-0.1t}$. Find the expected percent of radioactive material in the milk after the Chernobyl accident. Write answers rounded to the nearest tenth of a percent.

a. 0 days
b. 1 day
c. 10 days
d. 30 days
e. 100 days
a. Make a table of additional points and sketch a graph of $y = 100(2.7)^{-0.1t}$



23) An accidental spill of 75 grams of radioactive material in a local stream has led to the presence of radioactive debris decaying at a rate of 4% each day. Find how much debris still remains after 14 days. Use $y = 75(2.7)^{-0.04t}$ Round to the nearest tenth of a gram.

Math 70 . Exponential Functions, Exponential Equations, and Word Problems

Objectives:

1. Graph exponential functions.
2. Solve exponential equations algebraically.
3. Solve word problems that result in exponential equations.

1) Evaluate $f(x) = 2^x$ to complete the following table:

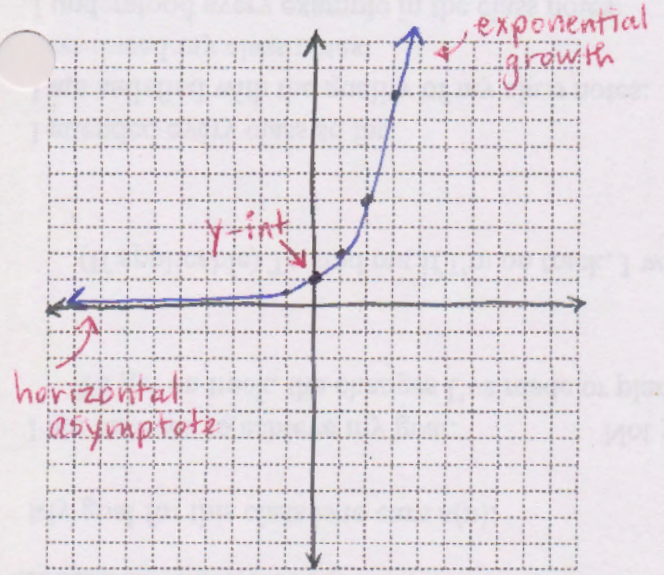
x	$f(x) = 2^x$
-4	$1/16$
-3	$1/8$
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8
4	16

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

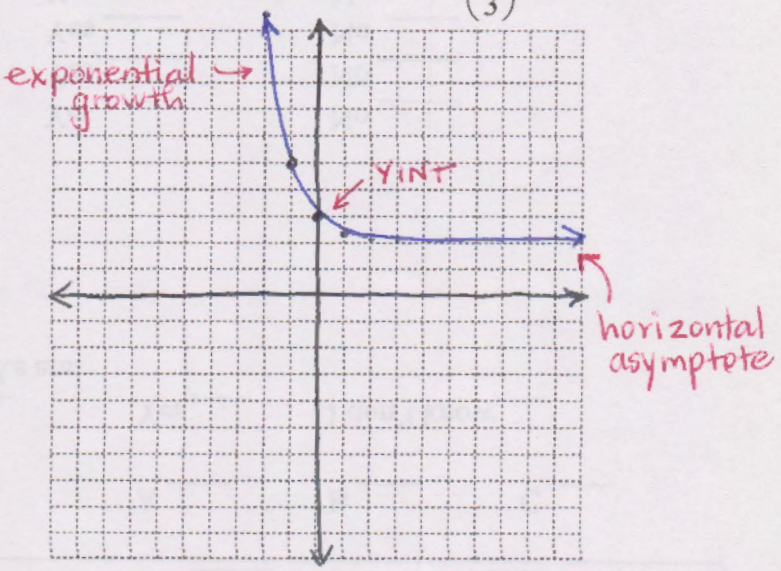
$2^0 = 1$

x	$f(x) = \left(\frac{1}{3}\right)^x + 2$
-4	83
-3	29
-2	11
-1	5
0	3
1	$2\frac{1}{3}$
2	$2\frac{1}{9}$
3	$2\frac{1}{27}$
4	$2\frac{1}{81}$

2) Sketch the graph of $f(x) = 2^x$



3) Sketch the graph of $f(x) = \left(\frac{1}{3}\right)^x + 2$



When sketching the graphs of exponential functions:

1. Plot enough points to show exponential growth. (extend to edge)
2. Plot the y-intercept
3. Plot enough points to show the horizontal asymptote. (extend to edge)

CAUTION: DON'T CHANGE THE SCALE TO SOMETHING > 1 OR YOUR Y-INT WILL BE UNCLEAR

Solve the equation.

step 1: isolate exponential (base & exponent)
step 2: rewrite with same base, simplify exponents.
step 3: ignore bases, set exponents equal.
[More detailed solutions at end of notes]

4) $2^x = 16$ $2^x = 2^4$ $\boxed{x=4}$

5) $3 \cdot 2^x = 48$ $2^x = \frac{48}{3} \Rightarrow 2^x = 16 \Rightarrow 2^x = 2^4 \Rightarrow \boxed{x=4}$

6) $2^{3x+1} = 16$ $2^{3x+1} = 2^4 \Rightarrow 3x+1 = 4 \Rightarrow 3x = 3 \Rightarrow \boxed{x=1}$

7) $2 \cdot 9^x = 54$ $9^x = \frac{54}{2} \Rightarrow 9^x = 27 \Rightarrow (3^2)^x = 3^3 \Rightarrow 3^{2x} = 3^3 \Rightarrow 2x = 3 \Rightarrow \boxed{x = \frac{3}{2}}$

8) $4^{x+3} = 8^x$ $(2^2)^{x+3} = (2^3)^x \Rightarrow 2^{2x+6} = 2^{3x} \Rightarrow 2x+6 = 3x \Rightarrow \boxed{x=6}$

9) $\frac{5}{4} = 5 \cdot 2^{3x} \Rightarrow \frac{5}{4} \cdot \frac{1}{5} = 2^{3x} \Rightarrow \frac{1}{4} = 2^{3x} \Rightarrow 2^{-2} = 2^{3x} \Rightarrow -2 = 3x \Rightarrow \boxed{x = -\frac{2}{3}}$

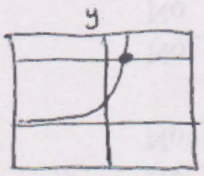
↑ exponential on RHS!

10) Use GC to estimate the solution to $5^x = 10$

$Y_1 = 5\text{^}X$ $Y_2 = 10$

WINDOW YMAX=15

2nd TRACE CALC 5: Intersect



$\boxed{x \approx 1.43}$

11) Find the amount owed at the end of 5 years if \$3000 is loaned at a rate of 10% compounded quarterly.

compound interest formula.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = ?$
$P = 3000$
$r = .10$
$n = 4$ times per year
$t = 5$ years

$A = 3000\left(1 + \frac{.1}{4}\right)^{(4 \cdot 5)}$

$A \approx 4915.849$

$\boxed{\$4915.85}$

12) An unusually wet spring has caused the size of the Cape Cod mosquito population to increase by 8% each day. If an estimated 200,000 mosquitoes are on Cape Cod on May 12, find how many mosquitoes will inhabit the Cape on May 25. Use $y = 200{,}000(2.7)^{0.08t}$. Round to the nearest thousand.

t = number of days.

May 25
− May 12
13 days

substitute $t = 13$

$y = 200000(2.7)^{(0.08 * 13)}$

$y = 561{,}886.1$

⇑ thousands place

$y = \boxed{562{,}000 \text{ mosquitos}}$

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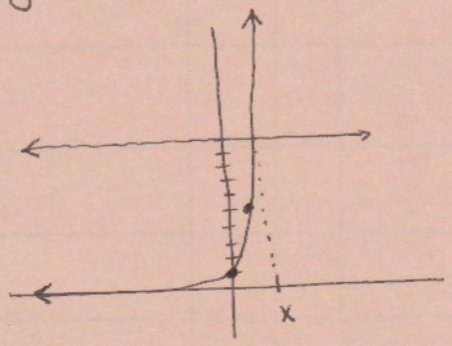
(10) Use GC to estimate solution to $5^x = 10$.
Why can't we solve this exactly (yet)?

$Y_1 = 5^x$

$Y_2 = 10$

WINDOW Ymax = 12

2nd TRACE = CALC
5. Intersect ENTER ENTER ENTER



$X \approx 1.4306766$

$\boxed{X \approx 1.43}$

This means that $5^1 = 5$ is less than 10
but $5^2 = 25$ is greater than 10.

We need an exponent value which is not a whole number.
To do this exactly, we will use logarithms in 9.8.

Recall:

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

A = amount at end including principal, interest and interest on interest.

P = principal or initial amount

r = interest rate, % written as decimal.

n = # times compounded per year

t = time in years.

Notice that this formula is an exponential if A(t) and P, r, n known.

ex: $P = 5000$
$r = 0.05$
$n = 12$

$$A(t) = 5000 \cdot \left(1 + \frac{.05}{12}\right)^{12t}$$

$12t$ exponent.

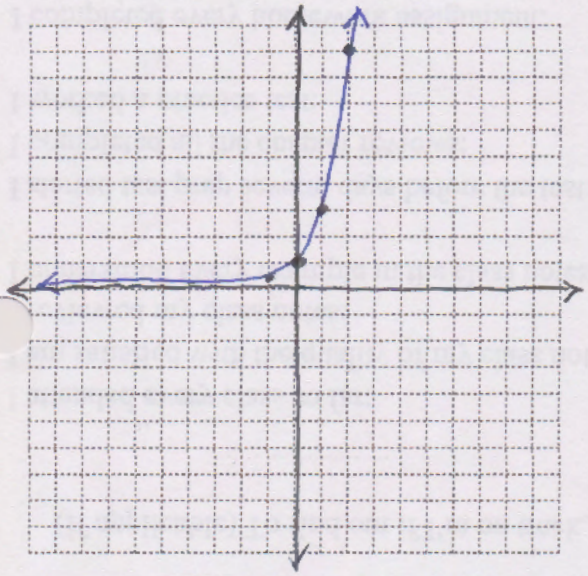
$$A(t) = \underbrace{5000}_{\text{coefficient}} \cdot \underbrace{\left(\frac{241}{240}\right)^{12t}}_{\text{base}}$$

EXTRAS:

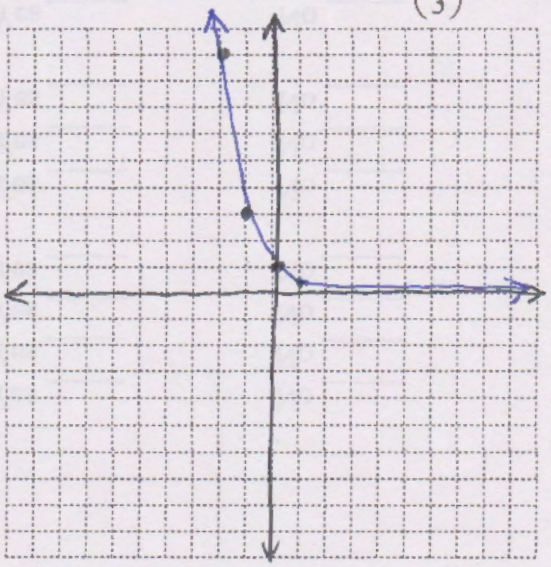
13) Evaluate $f(x) = 3^x$ to complete the following table:

x	$f(x) = 3^x$
-4	$1/81$
-3	$1/27$
-2	$1/9$
-1	$1/3$
0	1
1	3
2	9
3	27
4	81

14) Sketch the graph of $f(x) = 3^x$

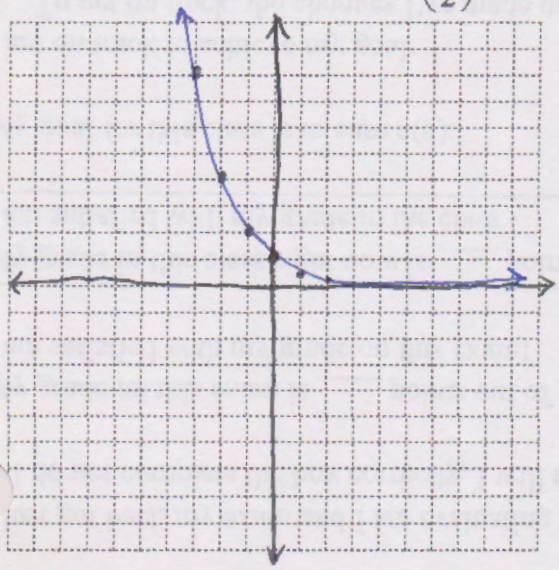


15) Sketch the graph of $f(x) = \left(\frac{1}{3}\right)^x$



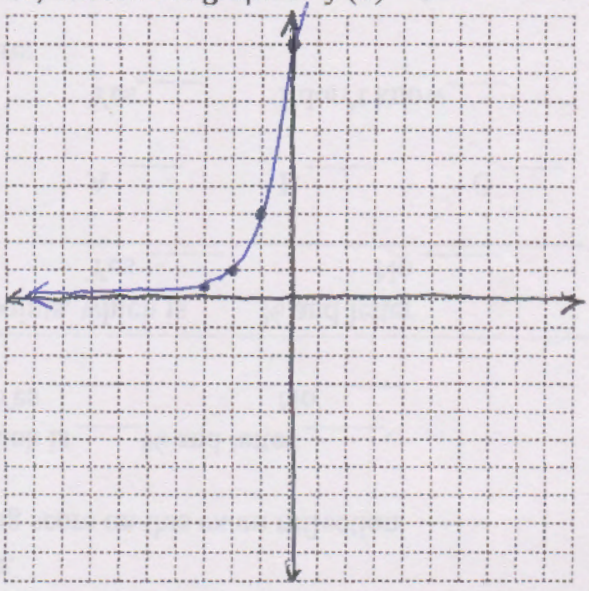
x	$\left(\frac{1}{3}\right)^x$
-4	81
-3	27
-2	9
-1	3
0	1
1	$1/3$
2	$1/9$
3	$1/27$
4	$1/81$

16) Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$



x	f(x)
-4	16
-3	8
-2	4
-1	2
0	1
1	$1/2$
2	$1/4$
3	$1/8$
4	$1/16$

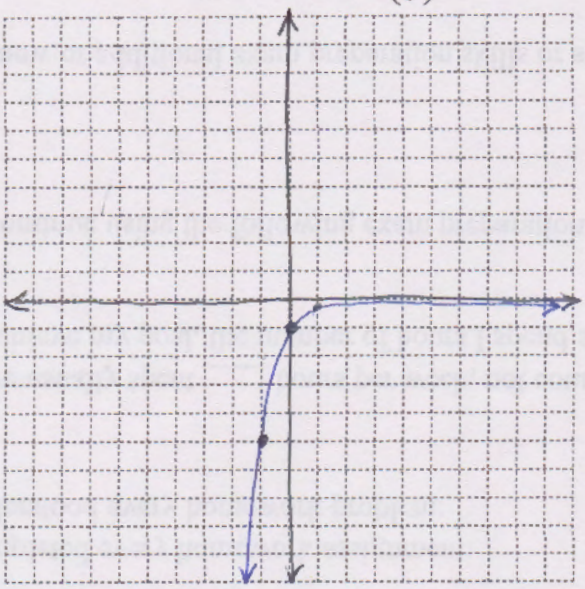
17) Sketch the graph of $f(x) = 3^{x+2}$



x	f(x)
-4	$1/9$
-3	$1/3$
-2	1
-1	3
0	9
1	27
2	81
3	way too big
4	

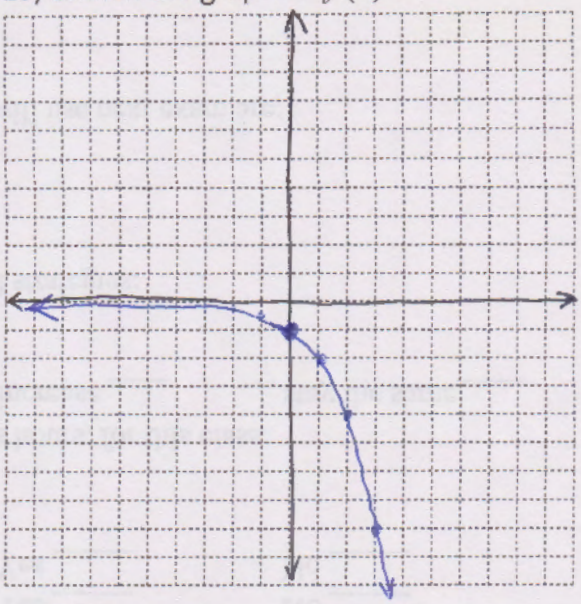
exponent before multiplying

18) Sketch the graph of $f(x) = -\left(\frac{1}{5}\right)^x$



x	f(x)
-4	too small
-3	-125
-2	-25
-1	-5
0	-1
1	$-\frac{1}{5}$
2	$-\frac{1}{25}$
3	$-\frac{1}{125}$
4	too close to 0

19) Sketch the graph of $f(x) = -2^x$



x	f(x)
-4	$-\frac{1}{16}$
-3	$-\frac{1}{8}$
-2	$-\frac{1}{4}$
-1	$-\frac{1}{2}$
0	-1
1	-2
2	-4
3	-8
4	-16

Solve the equation.

20) $25^x = \frac{1}{5} \Rightarrow (5^2)^x = 5^{-1} \Rightarrow 5^{2x} = 5^{-1} \Rightarrow 2x = -1$ $\boxed{x = -\frac{1}{2}}$

21) $4^{3x-7} = 32^{-2} \Rightarrow (2^2)^{3x-7} = (2^5)^{-2} \Rightarrow 2^{6x-14} = 2^{-10}$

$6x - 14 = -10 \Rightarrow 6x = 4$ $\boxed{x = \frac{2}{3}}$

22) As a result of the Chernobyl nuclear accident, radioactive debris was carried through the atmosphere, affecting cows and the milk they provide. The percent y of radioactive material in raw milk after t days is estimated by $y = 100(2.7)^{-0.1t}$. Find the expected percent of radioactive material in the milk after the Chernobyl accident. Give answers rounded to the nearest tenth of a percent.

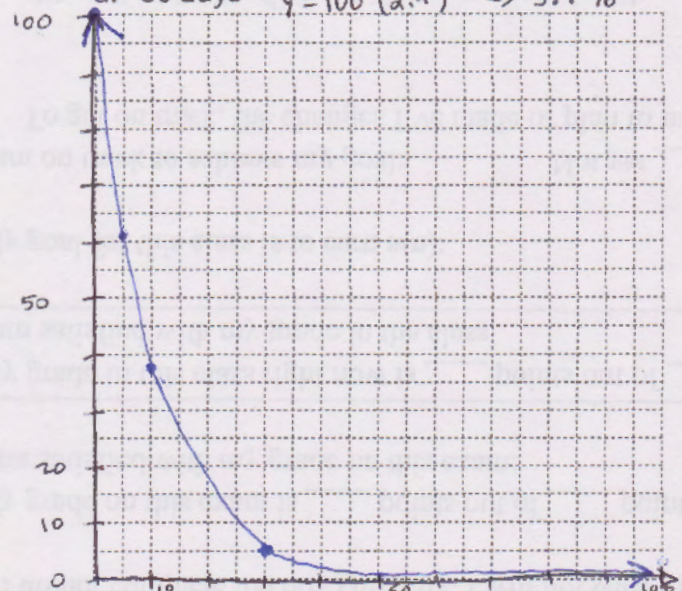
a. 0 days $y = 100(2.7)^0 \Rightarrow 100\%$

b. 1 day $y = 100(2.7)^{-0.1} \Rightarrow 90.5\%$

c. 10 days $y = 100(2.7)^{(-0.1)(10)} \Rightarrow 37.0\%$

d. 30 days $y = 100(2.7)^{-3} \Rightarrow 5.1\%$

e. 100 days $y = 100(2.7)^{-10} \Rightarrow 0.0\%$ (rounded)



a. Make a table of additional points and sketch a graph of $y = 100(2.7)^{-0.1t}$

x	y
5	60.9%
50	0.7%
75	0.1%

23) An accidental spill of 75 grams of radioactive material in a local stream has led to the presence of radioactive debris decaying at a rate of 4% each day. Find how much debris still remains after 14 days.

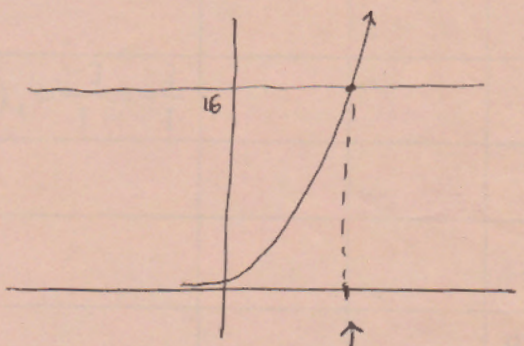
Use $y = 75(2.7)^{-0.04t}$ Round to the nearest tenth of a gram. $y = 75(2.7)^{(-.04*14)} = \boxed{43.0 \text{ grams}}$

Math 70

Recall: x^2 exponent 2, base x = "quadratic"
This is a type of polynomial.
Quadratic equations can be solved by factoring, CTS, QF.

2^x exponent x, base 2 = "exponential"
This is not a polynomial, and cannot be solved by factoring or CTS or QF.
Need an entirely new method.

✓ (4) Solve $2^x = 16$
a) using GC
b) using algebra



This x-value is the solution.
x=4

a) GC $y_1 = 2\wedge X$
WINDOW YMAX=20
$y_2 = 16$

2nd TRACE = CALC
5. Intersect ENTER ENTER ENTER

solution $x=4$
check: $2^4 = 16$.

b) Algebra.

step 1: Find a number which can be used as the base of the exponential on both LHS and RHS. This is called the common base. It is usually smaller than (or equal to) the given numbers. Rewrite LHS and RHS using this base.

base 2:
$2^x = 2^4$

step2: Since the bases are now equal, the exponents must be equal.

$x=4$

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✓ (5) Solve $3 \cdot 2^x = 48$

step 0: Isolate the exponential 2^x before finding a common base.

$\frac{3 \cdot 2^x}{3} = \frac{48}{3}$ divide both sides by 3

$2^x = 16$ now it's the same problem as (1)

$2^x = 2^4$

$\boxed{x = 4}$

* Extra $3 \cdot 2^x + 5 = 53$

✓ (6) Solve $2^{3x+1} = 16$

$2^{3x+1} = 2^4$ common base

$3x+1 = 4$ set exponents equal

$3x = 3$ isolate x.

$\boxed{x = 1}$

✓ (7) Solve $2 \cdot 9^x = 54$

Isolate exponential

$9^x = 27$

$9^2 = 81$ is too big.
but $9 = 3^2$ } so common base is 3.
and $27 = 3^3$ }

$(3^2)^x = (3^3)$ substitute equivalent exponential expressions using common base.

$3^{2x} = 3^3$ use exponent laws to simplify $(a^n)^m = a^{n \cdot m}$

$2x = 3$ set exponents equal

$\boxed{x = \frac{3}{2}}$ isolate x

✓ (8) Solve $4^{x+3} = 8^x$

$4^2 = 16$ is too big.
but $2^2 = 4$ } so common base is 2.
and $2^3 = 8$ }

$(2^2)^{x+3} = (2^3)^x$ substitute equivalent exponential expressions using common base

$2^{2x+6} = 2^{3x}$ use exponent laws to simplify.

$2x+6 = 3x$ set exponents equal

$\boxed{6 = x}$ isolate x

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✓ (9) Solve $\frac{5}{4} = 5 \cdot 2^{3x}$

Isolate exponential.
Remember that $\frac{1}{n} = n^{-1}$

$\frac{1}{4} = 2^{3x}$

So $\frac{1}{4} = 4^{-1}$

$4^{-1} = 2^{3x}$

while it's not a necessary step to do only this, it can be helpful.

We still need a common base

$2 = 2^1$
$4 = 2^2$ } so 2 is our common base.

$(2^2)^{-1} = 2^{3x}$

$2^{-2} = 2^{3x}$

exponent laws

$-2 = 3x$

set exponents equal

$\boxed{x = -\frac{2}{3}}$

isolate x

x (20) Solve $25^x = \frac{1}{5}$

$\frac{1}{5} = 5^{-1}$
$25 = 5^2$ } common base is 5

$(5^2)^x = 5^{-1}$

$5^{2x} = 5^{-1}$

exponent laws

set exponents equal

$2x = -1$

isolate x.

$\boxed{x = -\frac{1}{2}}$

x (21) Solve $4^{3x-7} = 32^{-2}$

$4^2 = 16$ not quite
$16^2 = 256$ } common base is 2 ⇒ SMALLER

$(2^2)^{3x-7} = (2^5)^{-2}$

$4 = 2^2$
$32 = 2^5$

$2^{6x-14} = 2^{-10}$

exponent laws

$6x - 14 = -10$

set exponents equal

$6x = 4$

isolate x

$\boxed{x = \frac{2}{3}}$

reduce

Math 62 11.2 Exponential Equations

Approximate to nearest ten-thousandth using GC

(3) e

2nd LN 1 ENTER

$e^{\wedge}$ e^{1}

$= 2.71828$

$\approx \boxed{2.7183}$ ← approximate

e ← exact

e to 25 decimal places

$e \approx 2.7182818284590452353603\ldots$

As

(4) e^2

2nd LN 2 ENTER

$= 7.38905$

$\approx \boxed{7.3891}$ ← approximate

e^2 ← exact

e is an irrational number!

- its decimal does not terminate
- its decimal does not repeat
- if we need an exact answer, we will write an expression using e.

(5) e^3

$= 20.08553$

$\approx \boxed{20.0855}$ ← approximate

e^3 ← exact

(6) e^{-1}

$= .36787$

$\approx \boxed{.3679}$ ← approximate

e^{-1} ← exact

Solve.

(7) $e^x = e^3$ same base

$\boxed{x = 3}$ set exponents equal

(8) $e^4 = e^x \cdot e^x$ exponent laws

$e^4 = e^{x+x}$

$e^4 = e^{2x}$

$4 = 2x$

$\boxed{2 = x}$

A simpler question

$x^2 \cdot x^3 = x^{2+3}$

add exponents